ECONOMIC GROWTH WITH NONRENEWABLE AND SUSTAINABLE ENERGY

*Henry Thompson**

The utilization of capital machinery and equipment provides ample motivation to include energy input in economic growth. Energy, as the third input, introduces its own dynamics and increases the potential of factor price substitution to influence economic growth. Thompson (2012) shows optimal depletion of a nonrenewable energy resource, with price rising at the rate of the capital return, dampens economic growth. The slowdown is long term, as Irimia-Vladu and Thompson (2007) point out, noting that optimal depletion would take 80 years to double the price of oil and reduce proven reserves by half.

The present paper develops the transition toward sustainable energy building on the neoclassical growth framework. Sustainable energy as its own capital requires separate investment that lowers consumption or investment in capital. The neoclassical Inada condition implies both types of energy are necessary as seems reasonable for the foreseeable future. The two energy prices are treated separately given the rising nonrenewable price and the market clearing price of sustainable energy.

---

*Henry Thompson is an Emeritus Professor of Economics at Auburn University. His research focuses on extending the scope and depth of international trade theory. He has published research in over 150 academic articles ranked in the top 5% by Repec. His textbook *International Economics: Global Markets and Competition* with World Scientific is now in its 5th edition. He received his Ph.D. from the University of Houston in 1981 under Roy Ruffin. Thompson was a founding member of the International Economics and Finance Society. In energy economics, he publishes research and works as a consultant. He and his artist wife Madeline Simos regularly spend some months in Greece every year.

Acknowledgements: James Cassing provided an important suggestion and Alexi Thompson useful comments at the 2024 Economics Workshop in Ios, Greece.

The Journal of Energy and Development, Vol. 49, Nos. 1-2

Factor shares of income and price elasticities influence the paths of per capita income, factor prices, and nonrenewable energy per capita. Preliminary analysis of the capital-labor model reveals the familiar wage increase assumes weak cross price elasticities of substitution. Including nonrenewable energy input, the three inputs introduce elastic substitutes and complements leaving only a few definite directions of change. Cobb-Douglas production leads to some definite directions of change. The model including sustainable energy leads to only a few definite partial effects.

The analysis turns to simulations based on averages of the U.S. labor growth rate, saving rate, and factor shares over the past six decades. The starting levels of output, fixed capital assets, labor force, and energy Btu inputs are for 2022. A range of factor price elasticities are considered starting with the error correction estimates in Copeland and Thompson (2022) based on 64 years of data with fixed capital assets imbedding technological change when energy Btu input is included. Sustainable energy including nuclear and hydroelectric provides one fifth of total energy input as a starting point. Ranges of saving rates for capital and sustainable energy are considered. Under some assumptions, the share of sustainable energy increases by half over 50 years.

Section 1 develops capital-labor growth as a production model in terms of factor shares and factor price elasticities. Section 2 adds nonrenewable energy developing the three-factor model of production and growth. Section 3 adds sustainable energy for a general equilibrium involving four factors of production each with its own dynamics. The conclusion, in Section 4, includes implications of the results for energy policy.

1. Capital-Labor Production and Growth

Neoclassical growth in Solow (1956) and Swan (1956) starts with the per capita constant returns production function $y = y(k)$ where $y \equiv Y/L$, $k \equiv K/L$ with output Y and inputs of capital K and labor L. The present assumption is a constant labor growth rate $\lambda \equiv L'/L$ where $L' \equiv dL/dt$. A constant marginal propensity to save σ leads to capital growth $K' = \sigma Y$ disregarding depreciation. The capital/labor ratio k evolves according to $k' = K'/L - \lambda k = \sigma y - \lambda k$. Introducing percentage changes where $\hat{k} \equiv k'/k$,

$$\hat{k} = \sigma y/k - \lambda. \tag{1}$$

Income Y is exhausted by competitive factor markets in $Y = rK + wL$ where the capital return r and wage w equal their marginal products Y_K and Y_L. Income per capita $y = rk + w$ changes according to $y' = rk' + kr' + w'$. Converting to percentage changes in a production framework introduces factor shares $\theta_K \equiv rK/Y = rk/y$ and $\theta_L \equiv wL/Y = w/y$,

$$\hat{y} = \theta_K\left(\hat{k} + \hat{r}\right) + \theta_L\hat{w}, \tag{2}$$

that also follows from the necessary condition $\theta_K{}' + \theta_L{}' = 0$.

The cost minimizing input a_K of capital per unit of output leads to per capita capital utilization $k = a_K y$ with changes $k' = ya_K{}' + a_K y'$. Homothetic production implies $a_K(r, w)$ varies only with factor prices leading to $a_K{}' = K_r r' + K_w w'$ introducing partial derivatives $K_r \equiv \partial a_K/\partial r$ and $K_w \equiv \partial a_K/\partial w$. Assuming the unit cost function is homogeneous of degree one, the a_i are homogeneous of degree zero. Euler's theorem then implies $rK_r + wK_w = 0$ leading to the zero sum across factor price elasticities $\sigma_{Kr} + \sigma_{Kw} = 0$ where $\sigma_{Kw} \equiv \hat{a}_K/\hat{w} = (w/a_K)K_w$ with the similar own elasticity σ_{Kr}. In elasticity form, per capita input of capital then evolves according to equation (3),

$$\hat{k} = \sigma_{Kr}\hat{r} + \sigma_{Kw}\hat{w} + \hat{y}. \tag{3}$$

Full employment of labor $L = a_L Y$ reduces to $1 = a_L y$ in per capita terms leading to equation (4):

$$\sigma_{Lr}\hat{r} + \sigma_{Lw}\hat{w} + \hat{y} = 0, \tag{4}$$

where $\sigma_{Lw} = -\sigma_{Lr}$ by Euler's theorem.

The capital-labor growth model in elasticity form combines equations (3) and (4) with (2),

$$\begin{pmatrix} \sigma_{Kr} & \sigma_{Kw} & 1 \\ \sigma_{Lr} & \sigma_{Lw} & 1 \\ -\theta_K & -\theta_L & 1 \end{pmatrix}\begin{pmatrix} \hat{r} \\ \hat{w} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} \hat{k} \\ 0 \\ \theta_K\hat{k} \end{pmatrix} \tag{5}$$

Parameters and state variables determine $\hat{k}$ treated as exogenous each period. The negative determinant $\Delta = -(\sigma_{Kw} + \sigma_{Lr}) < 0$ leads to the unambiguous qualitative results $\hat{r} = (\theta_L(1 + \sigma_{Lr}) + \theta_K\sigma_{Kw})\hat{k}/\Delta < 0$ and $\hat{y} = -\sigma_{Lr}\hat{k}/\Delta > 0$. The direction of change for the wage is ambiguous in $\hat{w} = (\theta_K(\sigma_{Kw} - 1) + \theta_L\sigma_{Lr})\hat{k}/\Delta$. The condition $\sigma_{Kw} + (\theta_L/\theta_K)\sigma_{Lr} < 1$ for a rising wage $\hat{w} > 0$ limits the degree of factor price substitution as labor input per unit of output diminishes with the rising r. The sizes of all adjustments diminish with growth as $\hat{k}$ declines. Also, a lower saving rate σ or higher labor growth λ would lower $\hat{k}$ and the sizes of all adjustments .

Cobb-Douglas production implies $\sigma_{Kw} = \theta_L = -\sigma_{Kr}$ and $\sigma_{Lr} = \theta_K = -\sigma_{Lw}$ leading to $\Delta = -1$, $\hat{r} = -\theta_L(1 + 2\theta_K)\hat{k} < 0$, and $\hat{y} = \theta_K\hat{k} > 0$. A rising wage $\hat{w} = (1 - 2\theta_L)\theta_K$ requires $\theta_L < 1/2$.

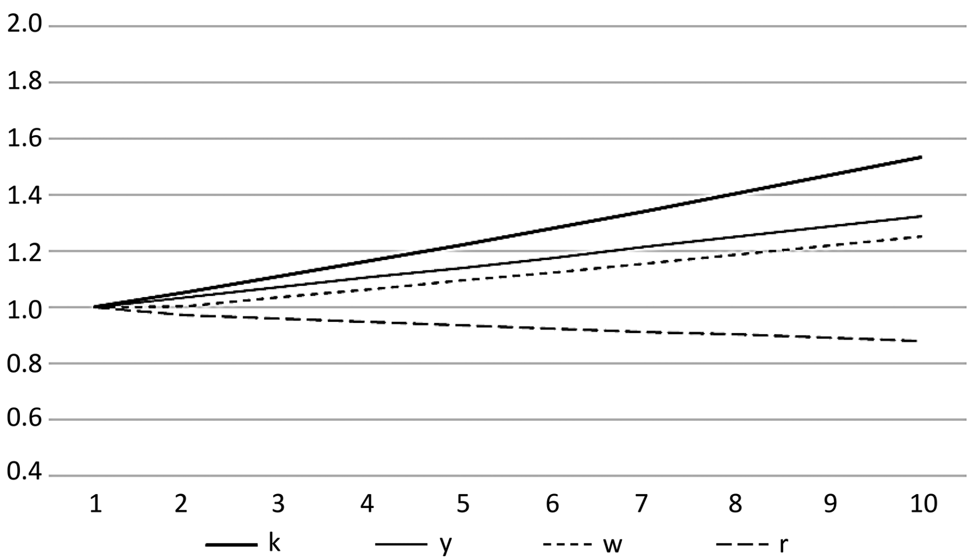
The simulations start with standard U.S. data in thousands from FRED and BEA for 2022 with y = 138, L = 167,000, and k = 563 based on fixed capital assets. Assuming the average labor share $\theta_L = 0.60$ over the past six decades implies w = 82.6. The residual capital share $\theta_K = 0.40$ and implies r = 0.0979. The average saving rate $\sigma = 0.30$ for private and public spending implies $\hat{k} = 0.063$ in (1). Labor growth is assumed constant at $\lambda = 0.01$.

Error correction estimates of factor price elasticities in Copeland and Thompson (2022) with 64 years of data parsimoniously add interaction terms for capital, labor, and Btu energy input to log linear production. These cross effects lead to moderately stronger elasticities involving labor and weaker elasticities for capital and energy relative to the log linear estimates. The present simulations rely on capital-energy interaction with the elasticities (σ_{Kw} σ_{Lr}) = (0.44 0.82). Simulations starting with Cobb-Douglas elasticities (0.60 0.40) are also considered.

Figure 1 shows growth paths over ten iterations with y rising 39%, k by 66%, and w by 30% as r falls −14%. This scale of adjustment is consistent with treating each period as five years based on the history of the 62% increase in k over five decades starting in 1970. The labor share θ_L falls slightly to 0.560 despite the rising wage and the 9% increase in L. Cross price elasticities weaken slightly over the iterations to (0.41, 0.75).

A simulation starting with Cobb-Douglas factor price elasticities leads to almost identical increases of 65% in k and 35% in y with a slightly larger increase in w of 33% and a larger −21% decrease in r. A simulation starting with hypothetical

Figure 1
KL SIMULATION



more elastic capital substitution favors capital input with slower growth in y and w. More elastic labor substitution leads to faster growth in y and less of a decline in r. Higher labor growth rates diminish $\hat{k}$ leading to somewhat weaker adjustments.

2. *Production and Growth with Nonrenewable Energy*

Optimal depletion of nonrenewable energy input E from the resource stock S where $E = -S'$ is developed in Dixit, Hammond, and Hoel (1980), Brock (1987), Hamilton (1995), Withagen and Asheim (1998), and Sato and Kim (2002). The energy/labor ratio $h \equiv E/L$ is added to the three-factor production function $y = f(k,h)$. Assuming zero marginal extraction cost, the Hotelling (1931) condition treating the nonrenewable energy stock S as an asset implies the energy price e rises at the rate of interest. Given capital input in dollar terms, the rate of increase in the price of energy equals the capital return in equation (6),

$$\widehat{e} = r. \tag{6}$$

The state of the economy including the endogenous energy input per capita h then determine $\hat{e}$ treated as exogenous to the periodic adjustments.

The adjustment in Income per capita $y = w + rk + eh$ expands to equation (7),

$$\widehat{y} = \theta_K\widehat{k} + \theta_E\widehat{h} + \theta_K\widehat{r} + \theta_L\widehat{w} + \theta_E\widehat{e} \tag{7}$$

This condition can be derived from $\theta_K{}' + \theta_L{}' + \theta_E{}' = 0$. Capital deepening $\hat{k} > 0$ has to more than offset the negative effect of depletion $\hat{h} < 0$ for economic growth.

Cost minimizing unit energy input $a_E(r,w,e)$ in $h = a_E y$ implies $h' = ya_E{}' + a_E y'$ leading to $a_E{}' = E_r r' + E_w w' + E_e e'$. In terms of factor price elasticities $\hat{a}_E = \sigma_{Er}\hat{r} + \sigma_{Ew}\hat{w} + \sigma_{Ee}\hat{e}$. Energy input per capita then evolves according to equation (8),

$$\widehat{h} = \sigma_{Er}\widehat{r} + \sigma_{Ew}\widehat{w} + \theta_{Ee}\widehat{e} + \widehat{y} \tag{8}$$

Similar expansions add $\sigma_{Ee}\hat{e}$ to (3) and $\sigma_{Le}\hat{e}$ to (4).

Elastic substitutes as well as complements are possible with three inputs as developed in Allen (1938), Takayama (1982, 1993), Jones and Easton (1983), and Thompson (1985, 2006). Cost minimization and Shephard's lemma lead to unit inputs a_i as partial derivatives c_i of the unit cost function c(r,w,e). Young's theorem implies symmetric substitution terms such as $E_r = \partial^2 c/\partial e\partial r = K_e$ implying the same signs for symmetric cross price elasticities. Negative second derivatives of the cost function imply negative own factor price elasticities. Homogeneity implies Euler's theorem $eE_e + rE_r + wE_w = 0$ leading to $\sigma_{Er} + \sigma_{Ew} = -\sigma_{Ee}$ with the zero sum across energy elasticities and similar conditions for capital and labor.

The condition $\sigma y > \lambda k$ in (1) for capital deepening $\hat{k} > 0$ provides a state variable along with the rising energy price $\hat{e} = r$ in (6). The first equation in the system (9) captures the adjustment in $\hat{k}$, the second the zero adjustment in per capita labor, and the third the endogenous $\hat{h}$ in (9). The fourth equation for $\hat{y}$ in (7) completes the system,

$$\begin{pmatrix} \sigma_{Kr} & \sigma_{Kw} & 0 & 1 \\ \sigma_{Lr} & \sigma_{Lw} & 0 & 1 \\ \sigma_{Er} & \sigma_{Ew} & -1 & 1 \\ -\theta_K & -\theta_L & -\theta_E & 1 \end{pmatrix} \begin{pmatrix} \hat{r} \\ \hat{w} \\ \hat{h} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} \hat{k} - \sigma_{Ke}\hat{e} \\ -\sigma_{Le}\hat{e} \\ -\sigma_{Ee}\hat{e} \\ \theta_K\hat{k} \end{pmatrix} \tag{9}$$

The qualitative directions of change depend upon factor price elasticities and factor shares as well as the rising capital-labor ratio and price of energy given by the state of the economy. The zero sums of factor price elasticities and unit sum of factor shares leave eight independent coefficients in the system matrix. Assuming concavity with a positive determinant Δ involves twelve product of elasticities and six more products involving factor shares as well.

The partial effects of the rising capital-labor ratio $\hat{k}$ include $\hat{r}/\hat{k} = -\theta_E(\theta_L + \theta_K(\sigma_{Kw} - \sigma_{Lw}) + \sigma_{Ew} - 2\sigma_{Lw})/\Delta < 0$. The expected increases $\hat{w}/\hat{k} > 0$ and $\hat{y}/\hat{k} > 0$ are weakly indicated in similar expressions but not necessary. The direction of change for $\hat{h}/\hat{k}$ is ambiguous consistent with the challenging empirical relationship between capital and energy in the literature.

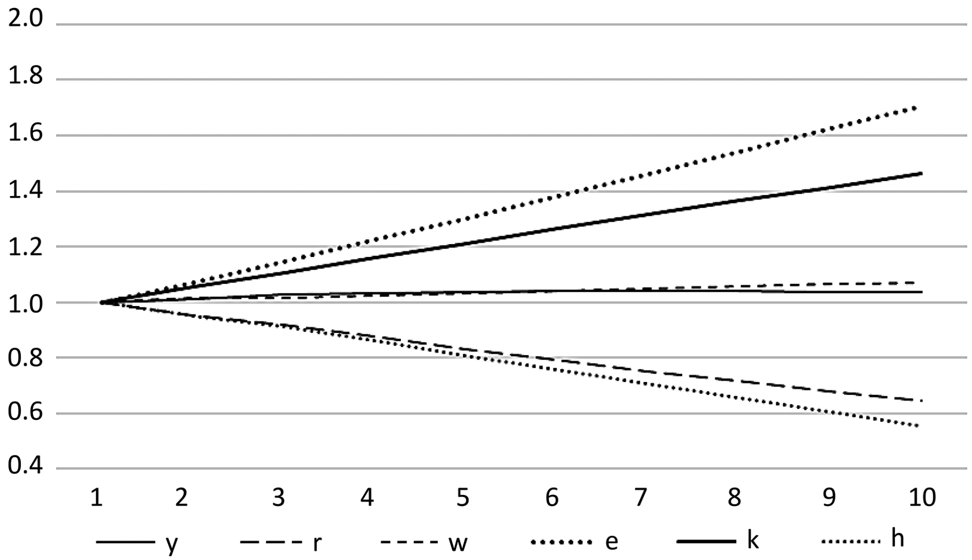
The partial effects of the rising price of energy $\hat{e} > 0$ include dampened economic growth in $\hat{y}/\hat{e} = (\theta_E - 1)(\sigma_{Ke}\sigma_{Lr} + \sigma_{Ke}\sigma_{Le} + \sigma_{Ke}\sigma_{Le})/\Delta < 0$. Energy input per capita falls as $\hat{h}/\hat{e} = (\theta_E - 1)\Sigma < 0$ where Σ refers to the sum of nine products of elasticities. There is a rising wage $\hat{w}/\hat{e} > 0$ if $\sigma_{Le} > \sigma_{Ke}$ that also favors a diminishing return to capital $\hat{r}/\hat{e} < 0$.

Cobb-Douglas production provides a guideline that would apply to moderate cross price elasticities starting with $\Delta = \theta_E(2 - \theta_E) > 0$. The partial effects of $\hat{k}$ are $\hat{r}/\hat{k} = -\theta_E(\theta_K + 2\theta_L)/\Delta < 0$, $\hat{w}/\hat{k} = \hat{h}/\hat{k} = \theta_K/\Delta > 0$, and $\hat{y}/\hat{k} = \theta_K\theta_E(2 - \theta_E)/\Delta > 0$. Two of these $\hat{k}$ effects are offset by the partial effects of $\hat{e}$ in $\hat{h}/\hat{e} = 2\theta_E(1 - \theta_E)/\Delta < 0$ and $\hat{y}/\hat{e} = -2\theta_E(1 - \theta_E)/\Delta < 0$ implying sizes of $\hat{k}$ and $\hat{e}$ determine the net directions of change in $\hat{h}$ and $\hat{y}$. The rising $\hat{e}$ has no partial effects on factor prices as $\hat{r}/\hat{e} = \hat{w}/\hat{e} = 0$ leaving the familiar wage increase and falling capital return.

Turning to simulations, total energy Btu input E = 97,209 in 2022 implies h = 592 combined with the data of the previous section. Assuming the average energy share $\theta_E = 0.10$ over the past four decades implies e = 0.0237. Energy input reduces the residual capital share to $\theta_K = 0.30$ and the capital return to r = 0.0734.

Figure 2 reports growth paths based on cross price elasticities ($\sigma_{Kw|Ke}$ $\sigma_{Lr|Le}$ $\sigma_{Er|Ew}$) = (0.22|0.22 0.50|0.31 0.40|0.23) and own elasticities (σ_{Kr} σ_{Lw} σ_{Ee}) = (−0.44 −0.81 −0.63) estimated with capital-labor interaction in Copeland and

Figure 2
KLE SIMULATION



Thompson (2022). Over ten iterations k increases 59% offsetting the decline of −39% in h and leading to an 11% increase in y well below 39% in KL model. The dampening effect of the 69% increase in e is apparent as $\hat{y}$ falls below 1% after period 5. The 17% increase in w is about half the KL model as r falls by more than twice at −38%. The labor share θ_L rises slightly to 0.630 as the other two fall. The cross price elasticities strengthen somewhat relative to the rising e trending to (0.18|0.27, 0.34|0.58, 0.43|0.47).

A hypothetical simulation with weak capital-energy complements in the cross price elasticities (0.30|−0.05 0.50|0.31 −0.05|0.40) leads to lower capital demand with k increasing 23% or less than half as much as in Figure 2. Energy per capita h rises 3% over the first three period before declining −6% overall compared to −39% in Figure 2. The increases of 8% in y and 15% in w are less than in Figure 2. The own labor elasticity σ_{Lw} becomes nearly elastic.

A simulation with elastic own energy $\sigma_{Ee} = -1.02$ in the cross price elasticities (0.21|0.26 0.35|0.58 0.40|0.62) based on the estimates with KL-KE interaction leads to a −3% decline in y with h falling −61% and r by −56%.

Summarizing, optimal depletion of nonrenewable energy leads to slower growth in y and w with a sharper decrease in r compared to the KL model as capital productivity diminishes. The labor share θ_L rises slightly as the rising K and e almost offset the falling r and E.

3. Introducing Sustainable Energy to Economic Growth

The present assumption is that sustainable energy input S is equivalent to its capital with its evolving price s determined along with the wage w and capital return r. Investment in S is assumed to depend on its exogenous marginal propensity to save γ in $S' = \gamma Y$.

Per capita input of sustainable energy $g \equiv S/L$ leads to $\hat{g} = \gamma y/g - \lambda > 0$ as a state variable along with $\hat{k}$ and $\hat{e}$. The fourth equation in the KLES system (10) accounts for $\hat{g}$. The endogenous vector includes the price adjustment $\hat{s}$ independent of the rising $\hat{e}$. The implicit assumption is that both types of energy input are required for cost minimizing production based on availability.

The system matrix includes twelve independent coefficient, presented in equation (10):

$$\begin{pmatrix} \sigma_{Kr} & \sigma_{Kw} & \sigma_{Ks} & 0 & 1 \\ \sigma_{Lr} & \sigma_{Lw} & \sigma_{Ls} & 0 & 1 \\ \sigma_{Er} & \sigma_{Ew} & \sigma_{Es} & -1 & 1 \\ \sigma_{Sr} & \sigma_{Sw} & \sigma_{Ss} & 0 & 1 \\ -\theta_K & -\theta_L & -\theta_S & -\theta_E & 1 \end{pmatrix} \begin{pmatrix} \hat{r} \\ \hat{w} \\ \hat{s} \\ \hat{h} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} \hat{k} - \sigma_{Ke}\hat{e} \\ -\sigma_{Le}\hat{e} \\ -\sigma_{Ee}\hat{e} \\ \hat{g} - \sigma_{Se}\hat{e} \\ \theta_K\hat{k} + \theta_S\hat{g} \end{pmatrix} \tag{10}$$

The complex concavity condition involves 33 triplet products of factor price elasticities along with 19 products weighted by factor shares. Qualitative analysis of the partial directions of change yields similar expressions for the three exogenous effects and no necessary qualitative results.

Cobb-Douglas production leads to a positive determinant Δ in a cubic expression of factor shares. The partial effects of $\hat{k}$ include the unambiguous $\hat{y} > 0$ and decreases for all three endogenous factor prices but an ambiguous effect on $\hat{h}$ as in the KLE model. The partial effects of $\hat{g}$ on factor prices are the opposite of $\hat{k}$ with a negative effect on $\hat{h}$. The partial effects of $\hat{e}$ on $\hat{y}$ and $\hat{h}$ are negative and on the three factor prices are likely negative as well. The positive $\hat{e}$, $\hat{k}$, and $\hat{g}$ leave no unambiguous directions of change.

Turning to simulations, sustainable Btu energy input S = 19,442, including nuclear and hydroelectric, accounts for 20% of total energy in 2022. Assuming prices of the two types of energy start at the same level, their shares $\theta_E = 0.08$ and $\theta_S = 0.02$ sum to 10% of Y and imply $e = s = 0.0237$. While investment in S would lower s, demand increases due to the rising nonrenewable price e. Simulations starting with $s > e$ lead to similar growth paths.

The estimated factor price elasticities σ_{Ee} and σ_{Ew} based on KE interaction are assumed for S in $\sigma_{Ss} = -0.63 = \sigma_{Ee}$ and $\sigma_{Sw} = 0.23 = \sigma_{Ew}$. Weak cross price elasticities are assumed between the two energy inputs in $\sigma_{Es} = \sigma_{Se} = 0.10$ implying $\sigma_{Er} = \sigma_{Sr} = 0.30$. The energy inputs are slightly weaker substitutes relative to r than in the KLE model. The various growth paths are not overly sensitive to these elasticities. The capital-labor KL elasticities (σ_{Kr} σ_{Kw} σ_{Lr} σ_{Lw}) are assumed at their KLE levels.

Saving rates σ and γ determine the growth of K for production and S for sustainable energy. The total saving rate is held at the same 30% of Y in the two simulations (σ γ) = (0.25 0.05) and (0.20 0.10). A third simulation assumes higher 40% total saving (σ γ) = (0.30 0.10). The resulting growth trends prove smooth with little acceleration and only a few changes in directions.

Figure 3 shows the paths for (σ γ) = (0.25 0.05) with growth slightly higher than the KLE model as y rises 12% even with the smaller 49% increase in k. Compared to KLE the 47% increase in g = S/L relieves depletion as h = E/L falls −33% compared to −36%. The wage w rises 21% compared to 17% as r falls half as much at −20%. The slower decrease in r leads to the steeper 78% increase in e compared to 69%. The share θ_S of sustainable energy doubles to 0.035 even as its price s rises 14%. The 62% increase in the weighted price of energy is below the 69% increase in e for KLE. This transition is consistent with historical k growth treating each iteration as five years.

Figure 4 shows growth paths for (σ γ) = (0.20 0.10) maintaining the same 30% total saving rate with higher S investment. The lower 13% growth of y follows from the diminished 38% growth in k. The wage w increases only 20% with the smaller −14% decline in r. The price s starts with a fall of −9% in the first period

Figure 3
KLES SIMULATION (σ γ) = (0.25 0.05)

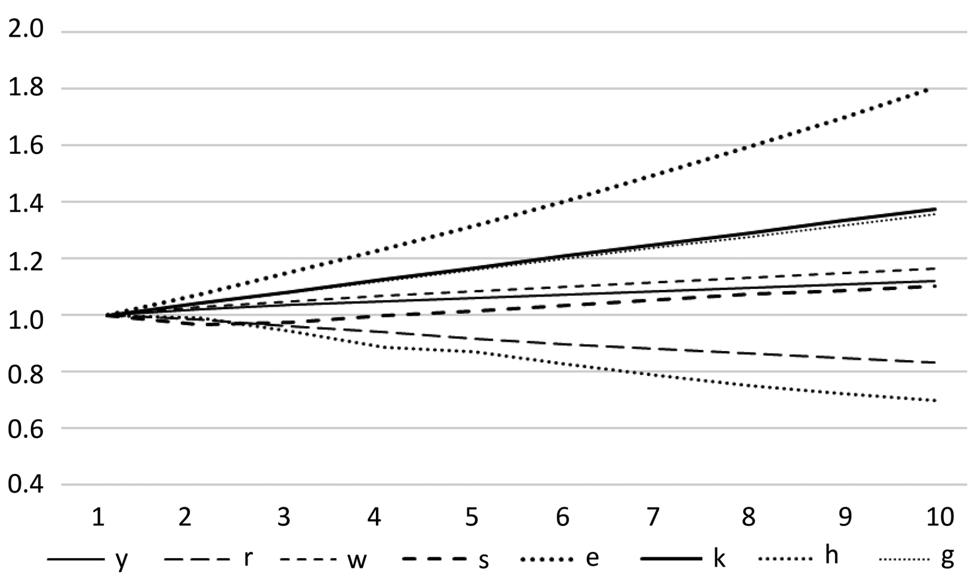
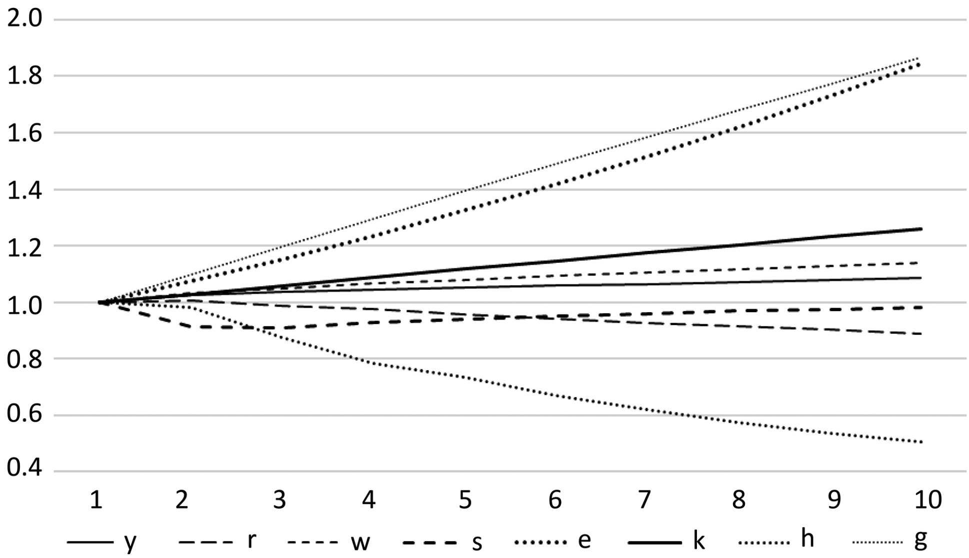


Figure 4
KLES SIMULATION (σ γ) = (0.20 0.10)



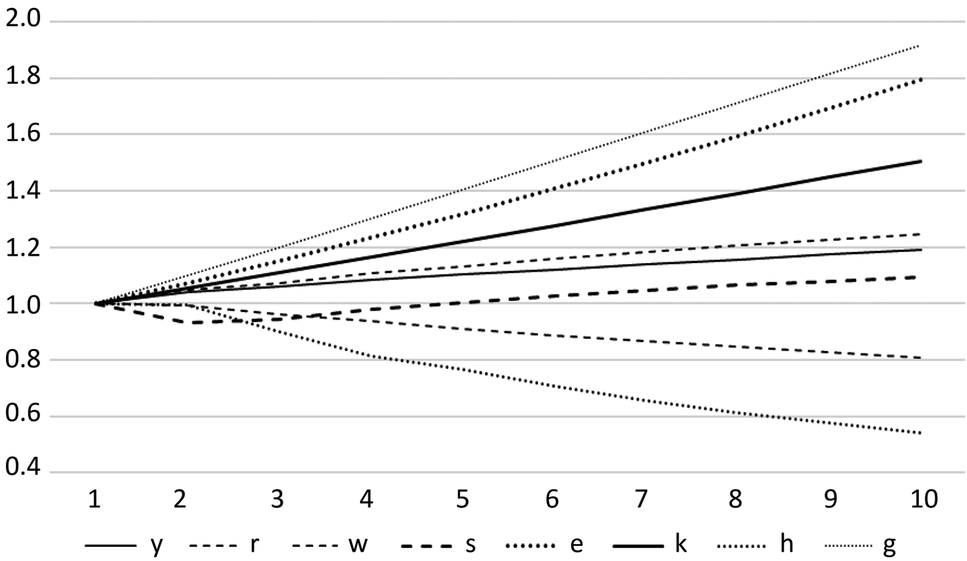
but slowly rises to finish at the same 14% with S providing half of total energy. The higher investment in S pays off with the lower 53% increase in the weighted energy price.

Figures 5 shows growth paths with higher overall saving (σ γ) = (0.30 0.10) leading to the largest increase in per capita income at 24%. Labor especially benefits with the 30% increase in w matching KL as the 63% increase in k approaches that level. Energy strongly shifts with S providing half and g more than doubling. The 53% increase in the weighted energy price increase is the same as (0.20 0.10).

Table 1 summarizes the results starting with high growth in the KL model without the rising price of nonrenewable or investment in sustainable energy. Consumption per capita c derived from income less total saving is about three times higher than in any simulation including energy. The KLE model leads to the smallest increases in w and c along with the largest decrease in r.

Across KLES simulations, lower S saving in (0.25 0.05) leads to the largest increase in s due to scarcity reflected by the smallest increase in g. The higher relative S saving (0.20 0.10) results in the smallest increase in s along with the largest increase in per capita consumption c and the largest decrease in h with S displacing E. The higher 40% total saving in (0.30 0.10) leads to the highest k, y, and w with the lowest weighted price of energy $z \equiv \theta_S s + \theta_E e$. Higher labor growth rates lead to similar results with smaller adjustments in factor prices up to small decreases in y and w.

Figure 5
KLES SIMULATION (σ γ) = (0.30 0.10)



Economic growth with only sustainable energy provides some perspective starting at S = 70,000 equal to the final level of E in the KLE simulation. The energy share $\theta_S = 0.10$ implies s = 0.0329. The 30% saving rate in (0.20 0.10) leads to smooth growth with increases of 38% in k and 22% in y. Energy per capita g increases 19% to a level consistent with total energy input across KLES simulations.

Table 1
SUMMARY OF SIMULATIONS

	KL (0.30)	**KLE (0.30)**	**KLES (0.25 .05)**	**KLES (0.20 0.10)**	**KLES (0.30 0.10)**
y	40%	11%	12%	14%	24%
r	−14%	−38%	−20%	−14%	−22%
c	28%	7.7%	8.4%	9.8%	9.6%
w	30%	17%	21%	20%	30%
e	-	74%	78%	83%	78%
s	-	-	16%	2%	12%
z	-	74%	62%	61%	53%
k	66%	58%	49%	37%	63%
h	-	−36%	−33%	−50%	−47%
g	-	-	47%	101%	106%

Income redistribution is relatively moderate with increases of 20% in w and 9% in s as r falls −7%. The 15% increase in c is only half the KL level but higher than any simulation including E. Higher investment in K in (0.30 0.10) leads to a larger 24% increase in y and a slightly lower 14% increase in c.

4. Conclusion

The present results reflect the critical role of energy in economic growth. Optimal depletion of nonrenewable energy slows economic growth due to its rising price and declining input. Investment in sustainable energy dampens these effects but lowers either consumption or investment in capital. The familiar rise in per capita income and wage depend on saving rates for sustainable energy versus capital. Factor price substitution influences growth and the distribution of income among capital, labor, and the two energy resources.

The present simulations based on the U.S. economy predict a smooth transition toward sustainable energy over the coming half century. Across a range of saving rates, sustainable energy per capita increases an average of 85% as its price rises 10%. The average 89% increase in the price of nonrenewable energy accompanies a reduction of −43% in its input per capita. The weighted price of energy increases 59% compared to the 74% increase with only nonrenewable energy. Income per capita increases 17% compared to 11% with only nonrenewable energy.

Regarding energy policy, the smooth economic transition suggests policymakers should rely on market competition. Examples of disruptive policies altering economic incentives include the oil depletion allowance, hydrocarbon targets, and subsidies for sustainable energy. Competitive markets will both conserve hydrocarbon resources and provide investment for sustainable energy.

REFERENCES

Allen, R. G. D. *Mathematical Analysis for Economists.* St. Martin's Press, 1938.

Brock, William. "Economic Dynamics: An Optimal Control Framework." *The New Palgrave*, edited by J. Eatwell, M. Milgate, and P. Newman, Macmillan, 1987, pp. 721–726.

Copeland, Cassandra, and Henry Thompson. "Energy Input Interaction in US Production." *Review of Economic Analysis*, 2022, pp. 525–541. DOI: 10.15353/rea.v14i4.3631.

Dixit, A., P. Hammond, and M. Hoel. "On Hartwick's Rule for Regular Maximin Paths of Capital Accumulation and Resource Depletion." *Review of Economic Studies*, vol. 47, 1980, pp. 551–556. http://hdl.handle.net/10.2307/2297306.

Hamilton, Kirk. "Sustainable Development, the Hartwick Rule, and Optimal Growth." *Environmental and Resource Economics*, vol. 5, 1995, pp. 393–411. http://hdl.handle.net/10.1007/BF00691576.

Hotelling, H. "The Economics of Exhaustible Resources." *Journal of Political Economy*, vol. 39, 1931, pp. 137–175. DOI: 10.1086/254195.

Irimia-Vladu, Marina, and Henry Thompson. "Oil Depletion and Terms of Trade." *Keio Economic Studies*, 2007, pp. 19–25.

Jones, Ron, and Stephen Easton. "Factor Intensities and Factor Substitution in General Equilibrium." *Journal of International Economics*, vol. 15, 1983, pp. 65–99. DOI: 10.1016/0022-1996(83)90042-9.

Sato, Ryuzo, and Youngduk Kim. "Hartwick's Rule and Economic Conservation Laws." *Journal of Economic Dynamics and Control*, vol. 26, 2002, pp. 437–449. DOI: 10.1016/S0165-1889(00)00055-5.

Solow, Robert. "A Contribution to the Theory of Economic Growth." *Quarterly Journal of Economics*, vol. 70, 1956, pp. 65–94. DOI: 10.2307/1884513.

Swan, Trevor. "Economic Growth and Capital Accumulation." *Economic Record*, vol. 32, 1956, pp. 234–261.

Takayama, Akira. "On Theorems of General Competitive Equilibrium of Production and Trade: A Survey of Recent Developments in the Theory of International Trade." *Keio Economic Studies*, vol. 19, 1982, pp. 1–38.

Takayama, Akira. *Analytical Methods in Economics.* University of Michigan Press, 1993.

Thompson, Henry. "Complementarity in a Simple General Equilibrium Production Model." *Canadian Journal of Economics*, vol. 18, 1985, pp. 616–621.

Thompson, Henry. "The Applied Theory of Energy Substitution in Production." *Energy Economics*, vol. 28, 2006, pp. 410–425. DOI: 10.1016/j.eneco.2005.01.005.

Thompson, Henry. "Economic Growth with a Nonrenewable Energy Resource." *Journal of Energy and Development*, vol. 36, 2012, pp. 35–43.